

20.5 MG road-maps for efficient statics and dynamics

A modern way to efficiently form static or dynamic equations with FBDs is to:¹

- Choose **scalar variables** that describe the relevant **unknown** configuration, motion, or forces.
- Complete the associated **MG road-maps** and **free-body diagrams**.²
- Complete the calculations specified by the **MG road-map equation**.

MG road-map for efficient statics and dynamics.

Variable	Translate/ Rotate	Direction (unit vector)	System <i>S</i>	FBD of <i>S</i>	About point	MG road-map equation	Additional Unknowns
				Draw			?
* If applicable: Additional constraint equations and their time-derivatives (e.g., closed linkages or rolling).							

Column	Enter the following information
1	Unknown scalar variable (e.g., a position, velocity, force, or torque variable).
2	Type of motion associated with the variable: translate or rotate .
3	Vector direction (e.g., unit vector \hat{u}) associated with the direction of motion.
4	List of objects whose motion (e.g., velocity or angular velocity) is directly effected by the variable in column 1 (“freeze” any variable other than the variable in column 1 and decide what objects must move). This picks a system S that reduces/eliminates constraint forces. Note: If the variable in column 1 is a force measure, treat it as a velocity measure and determine what objects move. If it is a torque measure, treat it as an angular velocity measure and determine what objects move.
5	Draw a free-body diagram of system S (draw relevant contact/distance forces). Note: See force/torque models for gravity, springs, dampers, air-resistance, etc., in Chapter 19.
6	If column 2 was rotate , choose a point <i>O</i> (or line <i>L</i>) about which moments are to be taken. Note: Choose point <i>O</i> to eliminate moments of unknown forces (e.g., contact forces on <i>S</i>) – look at FBD. Note: To facilitate calculations, you can slide the “about point” along the line <i>L</i> parallel to \hat{u} . This is because $\hat{u} \cdot \vec{M}^{S/O} = \hat{u} \cdot \vec{M}^{S/P}$ if both points <i>O</i> and <i>P</i> are on line <i>L</i> (proved in Section 17.1.3).
7a	If column 2 was translate , use: (<i>N</i> is a Newtonian reference frame) <div style="display: flex; justify-content: space-around; align-items: center; margin-top: 10px;"> <div style="border: 1px solid red; padding: 5px;"> $\hat{u} \cdot (\vec{F}^S = m^S * N\vec{a}^{S_{cm}})$ <p style="text-align: center; margin: 0;"><small>Dynamics</small></p> </div> <div style="font-size: 2em;">or</div> <div style="border: 1px solid red; padding: 5px;"> $\hat{u} \cdot \vec{F}^S = 0$ <p style="text-align: center; margin: 0;"><small>Statics</small></p> </div> </div> To calculate $m^S * N\vec{a}^{S_{cm}}$ for a system <i>S</i> of objects <i>A, B, C</i> : <div style="border: 1px solid green; padding: 5px; margin-top: 10px; text-align: center;"> $m^S * N\vec{a}^{S_{cm}} = m^A * N\vec{a}^{A_{cm}} + m^B * N\vec{a}^{B_{cm}} + m^C * N\vec{a}^{C_{cm}}$ <p style="margin: 0;"><small>(11.3)</small></p> </div> MotionGenesis™ command: <div style="display: flex; justify-content: space-around; align-items: center; margin-top: 10px;"> <div style="border: 1px solid red; padding: 2px;"> <code>Dot(u>, System.GetDynamics())</code> </div> <div style="font-size: 2em;">or</div> <div style="border: 1px solid red; padding: 2px;"> <code>Dot(u>, System.GetStatics())</code> </div> </div>
7b	If column 2 was rotate , use: <div style="display: flex; justify-content: space-around; align-items: center; margin-top: 10px;"> <div style="border: 1px solid red; padding: 5px;"> $\hat{u} \cdot (\vec{M}^{S/O} = \frac{N_d N \vec{H}^{S/O}}{dt} + \dots)$ <p style="text-align: center; margin: 0;"><small>(20.4)</small></p> </div> <div style="font-size: 2em;">or</div> <div style="border: 1px solid red; padding: 5px;"> $\hat{u} \cdot \vec{M}^{S/O} = 0$ <p style="text-align: center; margin: 0;"><small>Statics</small></p> </div> </div> For a system <i>S</i> with a particle <i>Q</i> and rigid body <i>B</i> : <div style="display: flex; justify-content: space-between; margin-top: 10px;"> <div style="border: 1px solid green; padding: 5px; width: 30%;"> $N\vec{H}^{S/O} = N\vec{H}^{Q/O} + N\vec{H}^{B/O}$ </div> <div style="border: 1px solid green; padding: 5px; width: 60%;"> <p>For particle <i>Q</i> $N\vec{H}^{Q/O} = \vec{r}^{Q/O} \times m^Q N\vec{v}^Q$ <small>(10.3)</small></p> <p>For rigid body <i>B</i> $N\vec{H}^{B/O} = N\vec{H}^{B/B_p} + \vec{r}^{B_p/O} \times m^B N\vec{v}^{B_{cm}}$ <small>(15.4)</small></p> <p>where: $N\vec{H}^{B/B_p} = \vec{I}^{B/B_p} \cdot N\vec{\omega}^B + \vec{r}^{B_{cm}/B_p} \times m^B N\vec{v}^{B_p}$ <small>(15.1)</small></p> </div> </div> MotionGenesis™ command: <div style="display: flex; justify-content: space-around; align-items: center; margin-top: 10px;"> <div style="border: 1px solid red; padding: 2px;"> <code>Dot(u>, System.GetDynamics(O))</code> </div> <div style="font-size: 2em;">or</div> <div style="border: 1px solid red; padding: 2px;"> <code>Dot(u>, System.GetStatics(O))</code> </div> </div>
8*	Additional constraint force/torque variables may appear in MG road-maps. If applicable, append configuration or motion constraints (e.g., closed linkages or rolling) that interrelate Column 1 position/velocity variables.

See MG road-map examples in next sections and at www.MotionGenesis.com ⇒ [Textbooks](#) ⇒ [Resources](#).

¹Mitiguy and Fregly invented **MG road-maps** to efficiently form static and dynamic equations. **MG road-maps** combine the simplicity of a spreadsheet with the physical insights of free-body diagrams. **MG road-maps** were inspired by the cleverness of D’Alembert and the mathematical rigor of Kane and Euler/Lagrange. **MG road-maps** are easy to teach/learn and approximate the efficiency of Kane’s method for most systems (except embedded constraints like rolling/gears).

²**Free-body diagrams (FBDs)** are a means to an end, not an end in itself. **MG road-maps** help determine **which FBDs** to draw and **what to do** with them, – which differs significantly from knowing **how** to draw FBDs.

20.5.1 MG road-map: Projectile motion (2D)

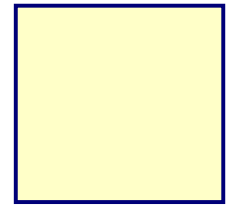
A baseball (particle Q) flies over Earth N (a Newtonian reference frame). Aerodynamic forces on the baseball are modeled as $-b\vec{v}$ (\vec{v} is Q 's velocity in N).

\hat{n}_x is horizontally-right, \hat{n}_y is vertically-upward, and N_o is home-plate (point fixed in N).



MG road-map for projectile motion x and y (\hat{n}_x, \hat{n}_y measures of Q 's position vector from N_o)

Variable	Translate/ Rotate	Direction (unit vector)	System S	FBD of S	About point	MG road-map equation
x	Translate	\hat{n}_x	Q	Draw	Not applicable	$\hat{n}_x \cdot (\vec{F}^Q = m^Q {}^N \vec{a}^Q)$ (20.1)
y	Translate	\hat{n}_y	Q	Draw	Not applicable	$\hat{n}_y \cdot (\vec{F}^Q = m^Q {}^N \vec{a}^Q)$ (20.1)
x	Dot(Nx >, Q.GetDynamics())					MotionGenesis command ©
y	Dot(Ny >, Q.GetDynamics())					MotionGenesis command ©



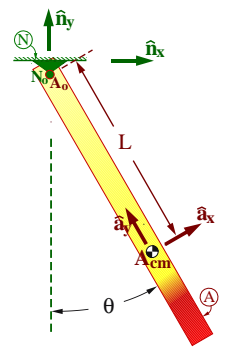
Draw FBD

Solution and simulation link at www.MotionGenesis.com ⇒ [Textbooks](#) ⇒ [Resources](#).

20.5.2 MG road-map: Rigid body pendulum (2D)

A non-uniform density rigid rod A is attached at point A_o of A by a frictionless revolute/pin joint to Earth N (Newtonian reference frame). The rod swings with a “pendulum angle” θ in a vertical plane that is perpendicular to unit vector \hat{a}_z .

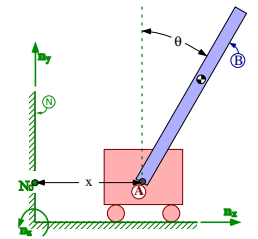
Variable	Translate/ Rotate	Direction (unit vector)	System S	FBD of S	About point	MG road-map equation
θ	Rotate	$\hat{a}_z = \hat{n}_z$	A	Draw	A_o	$\hat{a}_z \cdot (\vec{M}^{A/A_o} = \frac{{}^N d {}^N \vec{H}^{A/A_o}}{dt})$ (20.4)
θ	Dot(Az >, A.GetDynamics(A_o))					MotionGenesis command ©



Solution and simulation link at www.MotionGenesis.com ⇒ [Textbooks](#) ⇒ [Resources](#).

20.5.3 MG road-map: Inverted pendulum on cart (x and θ) (2D)

A rigid rod B is pinned to a massive cart A (modeled as a particle) that translates horizontally in a Newtonian reference frame N . The cart's position vector from a point N_o fixed in N is $x\hat{n}_x$ (\hat{n}_x is horizontally-right). B 's swinging motion in N is in a vertical plane perpendicular to \hat{n}_z (a unit vector fixed in both B and N).



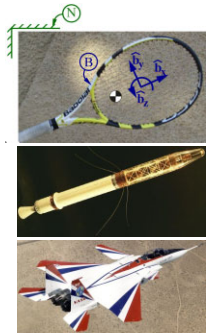
Variable	Translate/ Rotate	Direction (unit vector)	System S	FBD of S	About point	MG road-map equation
x	Translate	\hat{n}_x	A, B	Draw	Not applicable	$\hat{n}_x \cdot (\vec{F}^S = m^S * {}^N \vec{a}^{S_{cm}})$ (20.1)
θ	Rotate	$\hat{b}_z = \hat{n}_z$	B	Draw	A	$\hat{b}_z \cdot (\vec{M}^{B/A} = \frac{{}^N d {}^N \vec{H}^{B/A}}{dt} + \dots)$ (20.4)
x	Dot(Nx >, System(A,B).GetDynamics())					MotionGenesis command ©
θ	Dot(Bz >, B.GetDynamics(A))					MotionGenesis command ©

Homework 15.8 and Chapter 24 complete these calculations.

20.5.4 MG road-map: Rotating rigid body (3D)

Shown right is a rotating rigid body B (e.g., tennis racquet, spacecraft, or aircraft) in a Newtonian reference frame N . Right-handed orthogonal unit vectors $\hat{\mathbf{b}}_x, \hat{\mathbf{b}}_y, \hat{\mathbf{b}}_z$ are fixed in B .

Variable	Translate/ Rotate	Direction (unit vector)	System S	FBD of S	About point	<i>MG road-map equation</i>
ω_x	Rotate	$\hat{\mathbf{b}}_x$	B	Draw	B_{cm}	$\hat{\mathbf{b}}_x \cdot \left(\vec{\mathbf{M}}^{B/B_{cm}} = \frac{N d^N \vec{\mathbf{H}}^{B/B_{cm}}}{dt} \right)_{(20.4)}$
ω_y	Rotate	$\hat{\mathbf{b}}_y$	B	Draw	B_{cm}	$\hat{\mathbf{b}}_y \cdot \left(\vec{\mathbf{M}}^{B/B_{cm}} = \frac{N d^N \vec{\mathbf{H}}^{B/B_{cm}}}{dt} \right)_{(20.4)}$
ω_z	Rotate	$\hat{\mathbf{b}}_z$	B	Draw	B_{cm}	$\hat{\mathbf{b}}_z \cdot \left(\vec{\mathbf{M}}^{B/B_{cm}} = \frac{N d^N \vec{\mathbf{H}}^{B/B_{cm}}}{dt} \right)_{(20.4)}$
ω_x	Dot(\mathbf{B}_x >, B.GetDynamics(Bcm))					MotionGenesis command ☉
ω_y	Dot(\mathbf{B}_y >, B.GetDynamics(Bcm))					MotionGenesis command ☉
ω_z	Dot(\mathbf{B}_z >, B.GetDynamics(Bcm))					MotionGenesis command ☉

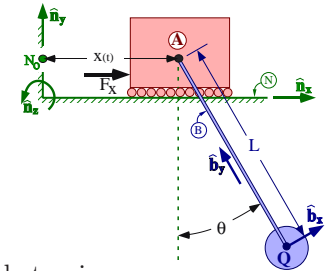


Solution and simulation link at www.MotionGenesis.com ⇒ [Textbooks](#) ⇒ [Resources](#).

Note: The “about point” is somewhat arbitrary. When B_{cm} is chosen: ${}^N \vec{\mathbf{H}}^{B/B_{cm}} = \vec{\mathbf{I}}^{B/B_{cm}} \cdot N \vec{\boldsymbol{\omega}}^B$. (15.2)

20.5.5 MG road-map: Bridge crane equations of motion (2D)

A payload (particle) Q is welded to the end of a light rigid cable B which swings in a Newtonian reference frame N . Cable B is pinned to a massive trolley A . Trolley A moves horizontally along a smooth slot fixed in N with a **specified** (known) displacement $x(t)$ due to a force of measure F_x (a linear actuator connects A to a point N_o of N).



MG road-map for pendulum angle θ , actuator force F_x , and cable tension

Variable	Translate/ Rotate	Direction (unit vector)	System S	FBD of S	About point	<i>MG road-map equation</i>
θ	Rotate	$\hat{\mathbf{n}}_z = \hat{\mathbf{b}}_z$	B, Q	Draw	A	$\hat{\mathbf{n}}_z \cdot \left(\vec{\mathbf{M}}^{S/A} = \frac{N d^N \vec{\mathbf{H}}^{S/A}}{dt} + \dots \right)$
F_x	Translate	$\hat{\mathbf{n}}_x$	A, B, Q	Draw	Not applicable	$\hat{\mathbf{n}}_x \cdot \left(\vec{\mathbf{F}}^S = m^S N \vec{\mathbf{a}}^{S_{cm}} \right)$
Tension	Translate	$\hat{\mathbf{b}}_y$	A, B, Q	Draw	Not applicable	$\hat{\mathbf{n}}_x \cdot \left(\vec{\mathbf{F}}^Q = m^Q N \vec{\mathbf{a}}^Q \right)$
θ	Dot(\mathbf{N}_z >, System(\mathbf{B}, Q).GetDynamics(\mathbf{A}))					MotionGenesis command ☉
F_x	Dot(\mathbf{N}_x >, System(\mathbf{A}, B, Q).GetDynamics())					MotionGenesis command ☉

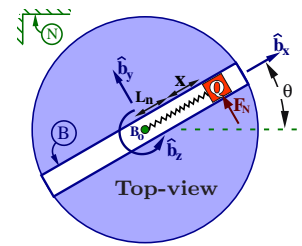
Student/Instructor version at www.MotionGenesis.com ⇒ [Textbooks](#) ⇒ [Resources](#)

Note: Only the θ road-map equation is needed to predict this system’s motion. The others are shown for illustrative purposes.

20.5.6 MG road-map: Particle on spinning slot (2D)

A particle Q slides on a straight slot B . The slot is connected with a revolute joint to a Newtonian frame N at point B_o so that B rotates in a horizontal plane perpendicular to $\hat{\mathbf{b}}_z$ ($\hat{\mathbf{b}}_z$ is vertically-upward and fixed in both B and N).

Note: Homework 14.7 completes the MG road-map calculations for x and θ .



MG road-map for x , θ , and F_N ($\hat{\mathbf{b}}_y$ measure of normal force on Q from B)

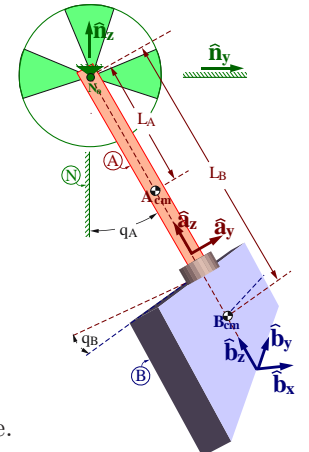
Variable	Translate/ Rotate	Direction (unit vector)	System S	FBD of S	About point	MG road-map equation
x	Translate	$\hat{\mathbf{b}}_x$	Q	Draw	Not applicable	$\hat{\mathbf{b}}_x \cdot (\vec{\mathbf{F}}^Q = m^Q {}^N \vec{\mathbf{a}}^Q)$
θ	Rotate	$\hat{\mathbf{b}}_z$	B, Q	Draw	B_o	$\hat{\mathbf{b}}_z \cdot (\vec{\mathbf{M}}^{S/B_o} = \frac{N d^N \vec{\mathbf{H}}^{S/B_o}}{dt})$
F_N	Translate	$\hat{\mathbf{b}}_y$	Q	Draw	Not applicable	$\hat{\mathbf{b}}_y \cdot (\vec{\mathbf{F}}^Q = m^Q {}^N \vec{\mathbf{a}}^Q)$
x	Dot(\mathbf{Bx} >,		Q.GetDynamics())		MotionGenesis command ©	
θ	Dot(\mathbf{Bz} >,		System(B, Q).GetDynamics(B_o))		MotionGenesis command ©	
F_N	Dot(\mathbf{By} >,		Q.GetDynamics())		MotionGenesis command ©	

Note: The F_N road-map equation is needed to predict motion if a friction force depends on μF_N .

20.5.7 MG road-map: Chaotic motion of a double pendulum (3D)

The schematic to the right shows a swinging babyboot attached by a shoelace to a rigid support. The mechanical model of the babyboot consists of a thin uniform rod A attached to a fixed support N by a revolute joint at point N_o and a uniform plate B connected to A with a second revolute joint at point B_o so B can rotate freely about A 's axis.

Note: The revolute joints' axes are *perpendicular*, not parallel.



Modeling considerations

- The plate, rod, and support are rigid.
- The revolute joints are ideal (massless, frictionless, no slop/flexibility).
- Earth is a Newtonian reference frame.
- Forces due to Earth's gravitation are uniform and constant.
- Other distance forces (electromagnetic and gravitational) and air-resistance are negligible.

Right-handed sets of unit vectors $\hat{\mathbf{n}}_x, \hat{\mathbf{n}}_y, \hat{\mathbf{n}}_z$; $\hat{\mathbf{a}}_x, \hat{\mathbf{a}}_y, \hat{\mathbf{a}}_z$; $\hat{\mathbf{b}}_x, \hat{\mathbf{b}}_y, \hat{\mathbf{b}}_z$ are fixed in N, A, B , respectively, with $\hat{\mathbf{n}}_x = \hat{\mathbf{a}}_x$ parallel to the revolute axis joining A to N , $\hat{\mathbf{n}}_z$ vertically-upward, $\hat{\mathbf{a}}_z = \hat{\mathbf{b}}_z$ parallel to the rod's long axis (and the revolute axis joining B to A), and $\hat{\mathbf{b}}_z$ perpendicular to plate B .

Complete the **MG road-map** for angles q_A (angle from $\hat{\mathbf{n}}_z$ to $\hat{\mathbf{a}}_z$ with $+\hat{\mathbf{n}}_x$ sense) and q_B (angle from $\hat{\mathbf{a}}_y$ to $\hat{\mathbf{b}}_y$ with $+\hat{\mathbf{a}}_z$ sense). Note: The "about point" can be shifted from B_o to B_{cm} since $\hat{\mathbf{b}}_z \cdot \vec{\mathbf{M}}^{B/B_{cm}} = \hat{\mathbf{b}}_z \cdot \vec{\mathbf{M}}^{B/B_o}$. (17.4)

Variable	Translate/ Rotate	Direction (unit vector)	System S	FBD of S	About point	MG road-map equation
q_A	Rotate	$\hat{\mathbf{a}}_x$	A, B	Draw	A_o	$\hat{\mathbf{a}}_x \cdot (\vec{\mathbf{M}}^{S/N_o} = \frac{N d^N \vec{\mathbf{H}}^{S/N_o}}{dt})$
q_B	Rotate	$\hat{\mathbf{b}}_z$	B	Draw	B_{cm}	$\hat{\mathbf{b}}_z \cdot (\vec{\mathbf{M}}^{B/B_{cm}} = \frac{N d^N \vec{\mathbf{H}}^{B/B_{cm}}}{dt})$
q_A	Dot(\mathbf{Ax} >,		System($\mathbf{A, B}$).GetDynamics(\mathbf{Ao}))		MotionGenesis command ©	
q_B	Dot(\mathbf{Bz} >,		\mathbf{B} .GetDynamics(\mathbf{Bcm}))		MotionGenesis command ©	

Student/Instructor version at www.MotionGenesis.com ⇒ [Textbooks](#) ⇒ [Resources](#)

20.5.9 MG road-map: Dynamicist on a turntable (ice-skater)

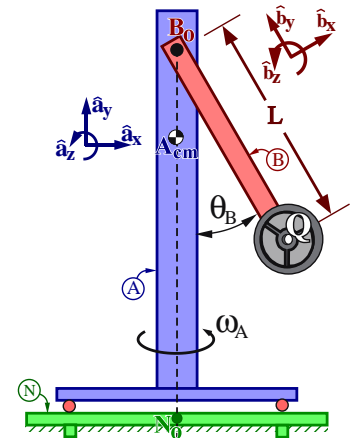
A dynamics instructor stands on a spinning turntable and swings a heavy dumbbell Q inward and outward to change his spin-rate (similar to the ice-skater). Q is modeled as a particle rigidly attached (welded) to the end of the instructor's hands.

The schematic (below-right) shows a rigid body A (modeling the instructor's legs, torso, and head) that rotates (without friction) relative to Earth (a Newtonian reference frame N) about a vertical axis that is fixed in both A and N and which passes through both point N_o of N and point A_{cm} (A 's center of mass).

A massless rigid arm B (modeling the instructor's arms and hands) attaches to A by a revolute motor (shoulder/muscles) whose revolute axis is horizontal and located at point B_o of B (B_o lies on the vertical axis connecting N_o and A_{cm}).

The motor (muscles) **specifies** B 's angle θ_B relative to A to change in a known (prescribed) manner from 0 to π rad in 4 seconds ($\theta_B = \pi \frac{t}{4}$).

Right-handed orthogonal unit vectors $\hat{\mathbf{a}}_x, \hat{\mathbf{a}}_y, \hat{\mathbf{a}}_z$ and $\hat{\mathbf{b}}_x, \hat{\mathbf{b}}_y, \hat{\mathbf{b}}_z$ are fixed in A and B , respectively, with $\hat{\mathbf{a}}_y$ vertically-upward, $\hat{\mathbf{b}}_z = \hat{\mathbf{a}}_z$ parallel to the revolute motor's axis, and $\hat{\mathbf{b}}_y$ directed from Q to B_o .



Quantity	Symbol	Type	Value
Earth's gravitational constant	g	Constant	$9.8 \frac{m}{s^2}$
Distance between Q and B_o	L	Constant	0.7 m
Mass of Q	m	Constant	12 kg
A 's moment of inertia about line $\overline{A_{cm} B_o}$	I_{yy}	Constant	0.6 kg m^2
Angle from $\hat{\mathbf{a}}_y$ to $\hat{\mathbf{b}}_y$ with $+\hat{\mathbf{a}}_z$ sense	θ_B	Specified	$0.25 \pi t$ rad
$\hat{\mathbf{a}}_y$ measure of A 's angular velocity in N	ω_A	Variable	

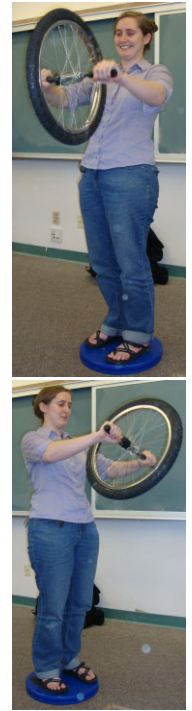
Complete the **MG road-map** for the turntable's "spin-rate" ω_A (Note: The "about point" is not unique)

Variable	Translate/ Rotate	Direction (unit vector)	System S	FBD of S	About point	MG road-map equation
ω_A	Rotate	$\hat{\mathbf{a}}_y$	A, B, Q	Draw	B_o	$\hat{\mathbf{a}}_y \cdot (\vec{M}^{S/B_o} = \frac{N_d^N \vec{H}^{S/B_o}}{dt})$
ω_A	Dot(\mathbf{Ay} , System($\mathbf{A, B, Q}$) .GetDynamics(\mathbf{Bo}))					MotionGenesis command ©

Student/Instructor version at www.MotionGenesis.com ⇒ [Textbooks](#) ⇒ [Resources](#)

20.5.10 MG road-map: Instructor on turntable with spinning wheel (3D)

The pictures to the right shows dynamicist Dr. G standing on a spinning turntable and holding a spinning bicycle wheel.



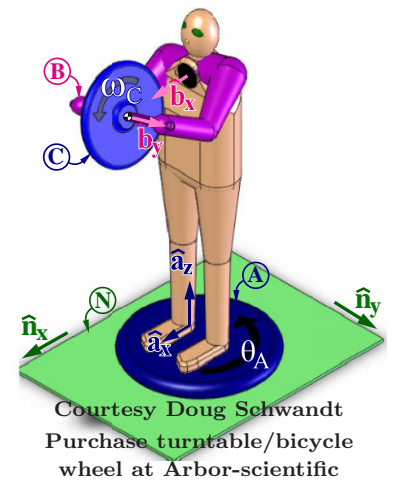
The mechanical model (below right) has a rigid body A (modeling the turntable, legs, torso, and head) that can freely rotate relative to Earth (Newtonian reference frame N) about a vertical axis that is fixed in both A and N and which passes through the center of the turntable (point N_o) and A_{cm} (A 's center of mass).

A light (massless) rigid frame B (modeling the shoulders, arms, hands, and a portion of the bicycle wheel's axle) is attached to A by a revolute motor at point B_o of B (B_o lies on the vertical axis passing through A_{cm}). The motor's revolute axis passes through points B_o and C_{cm} , is horizontal, and is parallel to $\hat{b}_x = \hat{a}_x$.

A rigid bicycle wheel C is attached to B by a frictionless revolute joint whose axis passes through C_{cm} (C 's center of mass) and is parallel to \hat{b}_y .

Right-handed orthogonal unit vectors $\hat{a}_x, \hat{a}_y, \hat{a}_z$ and $\hat{n}_x, \hat{n}_y, \hat{n}_z$ are fixed in A and N , respectively. Initially $\hat{a}_i = \hat{n}_i$ ($i = x, y, z$), and then rigid body A is subjected to a right-handed rotation characterized by $\theta_A \hat{a}_z$ where $\hat{a}_z = \hat{n}_z$ is directed vertically-upward and \hat{a}_x points from Dr. G's back to front (parallel to the axis of the revolute motor connecting A and B).

Unit vectors $\hat{b}_x, \hat{b}_y, \hat{b}_z$ are fixed in B . Initially $\hat{b}_i = \hat{a}_i$ ($i = x, y, z$), then B is subjected to a θ_B ($\hat{a}_x = \hat{b}_x$) right-handed rotation in A where \hat{b}_y is directed along the wheel's axle from Dr. G's right-to-left hand. Dr. G changes θ_B in a **specified** sinusoid manner with amplitude 30° and period 4 seconds.

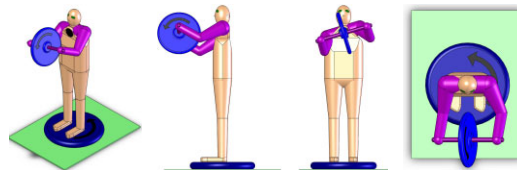


Quantity	Symbol and type		Value
Mass of C	m^C	Constant	2 kg
Distance between B_o and C_{cm}	L_x	Constant	0.5 m
A 's moment of inertia about B_o for \hat{a}_z	I_{zz}^A	Constant	0.64 kg m ²
C 's moment of inertia about C_{cm} for \hat{b}_x	I^C	Constant	0.12 kg m ²
C 's moment of inertia about C_{cm} for \hat{b}_y	J^C	Constant	0.24 kg m ²
Angle from \hat{n}_x to \hat{a}_x with $+\hat{n}_z$ sense	θ_A	Variable	
Angle from \hat{a}_y to \hat{b}_y with $+\hat{a}_x$ sense	θ_B	Specified	$\frac{\pi}{6} \sin(\frac{\pi}{2} t)$
\hat{b}_y measure of C 's angular velocity in B	ω_C	Variable	

Complete the **MG road-map** for θ_A and ω_C (the "about points" are not unique).

Variable	Translate/ Rotate	Direction (unit vector)	System S	FBD of S	About point	MG road-map equation
θ_A	Rotate	\hat{a}_z	A, B, C	Draw	B_o	$\hat{a}_z \cdot (\vec{M}^{S/B_o} = \frac{N d^N \vec{H}^{S/B_o}}{dt})$
ω_C	Rotate	\hat{b}_y	C	Draw	C_{cm}	$\hat{b}_y \cdot (\vec{M}^{C/C_{cm}} = \frac{N d^N \vec{H}^{C/C_{cm}}}{dt})$
θ_A	Dot($\langle \mathbf{Az} \rangle$, System($\langle \mathbf{A, B, C} \rangle$).GetDynamics($\langle \mathbf{Bo} \rangle$))					MotionGenesis command ©
ω_C	Dot($\langle \mathbf{By} \rangle$, System($\langle \mathbf{C} \rangle$).GetDynamics($\langle \mathbf{Ccm} \rangle$))					MotionGenesis command ©

Student/Instructor version at www.MotionGenesis.com ⇒ [Textbooks](#) ⇒ [Resources](#)



20.5.11 MG road-map: Bear riding a unicycle on a high-wire (3D)

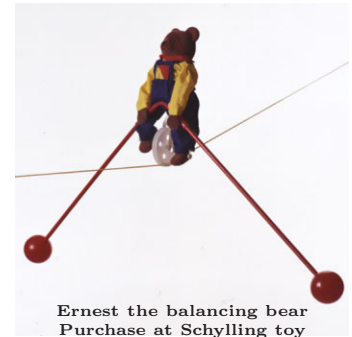
The figures to the right show a (massless) pulley-wheel B that **rolls** along a taut (rigid) cable N (fixed on Earth, a Newtonian frame). Rigid body C (seat, rider, and balancing poles) attach to B with an ideal revolute motor at B_o (B 's centroid). The motor axis is aligned with B 's symmetry axis.

Right-handed orthogonal unit vectors $\hat{n}_x, \hat{n}_y, \hat{n}_z$ are fixed in N with \hat{n}_z vertically-upward and \hat{n}_x directed horizontally along the cable from a point N_o (fixed in N) to B_N (B 's rolling point of contact with N).

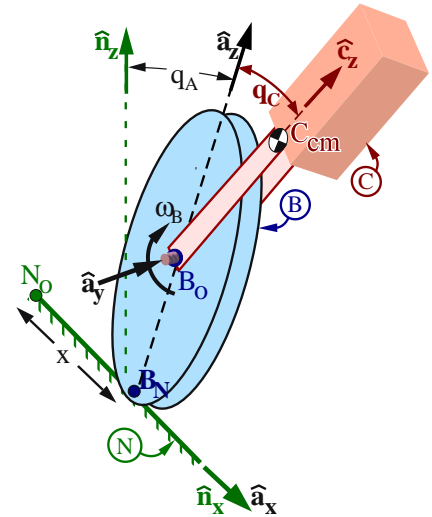
Right-handed orthogonal unit vectors $\hat{a}_x, \hat{a}_y, \hat{a}_z$ are directed with $\hat{a}_x = \hat{n}_x$, \hat{a}_y parallel to the motor axis, and \hat{a}_z from B_N to B_o .

Right-handed unit vectors $\hat{c}_x, \hat{c}_y, \hat{c}_z$ are parallel to C 's principal inertia axes about C_{cm} (C 's center of mass), with $\hat{c}_y = \hat{a}_y$ and \hat{c}_z from B_o to C_{cm} (with balancing poles, C_{cm} is below B_o and L_C is negative).

Quantity	Symbol	Type	Value
Earth's gravitational constant	g	Constant	9.8 m/s ²
Radius of B	r_B	Constant	30 cm
\hat{c}_z measure of C_{cm} 's position vector from B_o	L_C	Constant	-35 cm
Mass of C	m^C	Constant	2 kg
C 's moment of inertia about C_{cm} for \hat{c}_x	I	Constant	3.4 kg m ²
C 's moment of inertia about C_{cm} for \hat{c}_y	J	Constant	3.2 kg m ²
C 's moment of inertia about C_{cm} for \hat{c}_z	K	Constant	2.8 kg m ²
\hat{a}_y measure of motor torque on B from C	T_y	Specified	below
Angle from \hat{n}_z to \hat{a}_z with $-\hat{n}_x$ sense	q_A	Variable	
\hat{a}_y measure of ${}^A\omega^B$ (${}^A\omega^B = \omega_B \hat{a}_y$)	ω_B	Variable	
Angle from \hat{a}_z to \hat{c}_z with $+\hat{a}_y$ sense	q_C	Variable	
\hat{n}_x measure of \vec{r}^{B_N/N_o}	x	Variable	



Ernest the balancing bear
Purchase at Schylling toy



Form a complete set of **MG road-maps** for this systems's equations of motion (solution is not unique).

If necessary, add more **MG road-maps** so there are the same number of equations as unknowns.

Variable	Translate/Rotate	Direction (unit vector)	System S	FBD of S	About point	MG road-map equation	Additional Unknowns
q_A	Rotate	\hat{a}_x	A, B, C	Draw	B_N	$\hat{a}_x \cdot (\vec{M}^{S/B_N} = \frac{d}{dt} \vec{H}^{S/B_N} + \dots)$	
ω_B	Rotate	\hat{a}_y	B, C	Draw	B_N	$\hat{a}_y \cdot (\vec{M}^{S/B_N} = \frac{d}{dt} \vec{H}^{S/B_N} + \dots)$	
q_C	Rotate	\hat{a}_y	C	Draw	B_o	$\hat{a}_y \cdot (\vec{M}^{C/B_o} = \frac{d}{dt} \vec{H}^{C/B_o} + \dots)$	
x	Translate	\hat{a}_x	A, B, C	Draw	Not applicable	$\hat{a}_x \cdot (\vec{F}^S = m^S * N \vec{a}^{S_{cm}})$	* F_x

* Additional scalar constraint equation(s): $\dot{x} - r\omega_B = 0$

Instructor notes: One way to eliminate F_x from this analysis is to solve $\dot{x} = r\omega_B$ and eliminate the MG road-map for x . Alternate ω_B MG road-map (that allows for sliding): "System" B , "About point" B_o , "Additional unknown" F_x .

q_A	Dot($\langle \mathbf{Ax} \rangle$,	System($\langle \mathbf{A}, \mathbf{B}, \mathbf{C} \rangle$).GetDynamics($\langle \mathbf{B}_N \rangle$))	MotionGenesis command @
ω_B	Dot($\langle \mathbf{Ay} \rangle$,	System($\langle \mathbf{B}, \mathbf{C} \rangle$).GetDynamics($\langle \mathbf{B}_N \rangle$))	MotionGenesis command @
q_C	Dot($\langle \mathbf{Ay} \rangle$,	\mathbf{C} .GetDynamics($\langle \mathbf{B}_o \rangle$))	MotionGenesis command @
x	Dot($\langle \mathbf{Ax} \rangle$,	System($\langle \mathbf{A}, \mathbf{B}, \mathbf{C} \rangle$).GetDynamics())	MotionGenesis command @
	SolveDt($\mathbf{x}' - r*\omega_B = 0$,	\mathbf{x}')	MotionGenesis command @

To move the unicycle to $x_{\text{Desired}} = 10$ m, use a "PD control law" with $T_y = -0.3(x - x_{\text{Desired}}) - 0.6\dot{x}$.

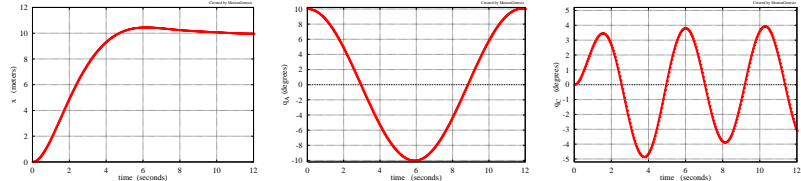
Optional simulation:

Plot x, q_A, q_C for $0 \leq t \leq 12$ sec.

Use initial values:

$$x = 0 \text{ m} \quad q_A = 10^\circ \quad \dot{q}_C = 0^\circ$$

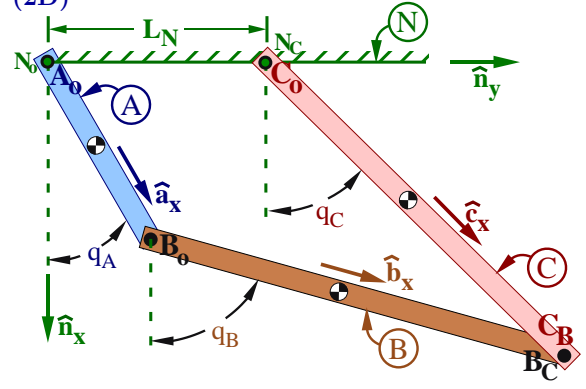
$$\dot{x} = 0 \quad \dot{q}_A = 0 \quad \dot{q}_C = 0$$



Solution at www.MotionGenesis.com ⇒ [Get Started](#) ⇒ Bear on rolling unicycle.

20.5.12 MG road-map: Four-bar linkage statics (2D)

The figure to the right shows a planar four-bar linkage consisting of frictionless-pin-connected uniform rigid links A, B, and C and ground N.



- Link A connects to N and B at points A_o and A_B
- Link B connects to A and C at points B_o and B_C
- Link C connects to N and B at points C_o and C_B
- Point N_o of N is coincident with A_o
- Point N_C of N is coincident with C_o

Right-handed orthogonal unit vectors $\hat{a}_i, \hat{b}_i, \hat{c}_i, \hat{n}_i$ ($i = x, y, z$) are fixed in A, B, C, N, with:

- \hat{a}_x directed from A_o to A_B
- \hat{b}_x directed from B_o to B_C
- \hat{c}_x directed from C_o to C_B
- \hat{n}_x vertically-downward
- \hat{n}_y directed from N_o to N_C
- $\hat{a}_z = \hat{b}_z = \hat{c}_z = \hat{n}_z$ parallel to pin axes

As in Hw 8.7, create the following “loop equation” and dot-product with \hat{n}_x and \hat{n}_y .

$$L_A \hat{a}_x + L_B \hat{b}_x - L_C \hat{c}_x - L_N \hat{n}_y = \vec{0}$$

Quantity	Symbol	Value
Length of link A	L_A	1 m
Length of link B	L_B	2 m
Length of link C	L_C	2 m
Distance between N_o and N_C	L_N	1 m
Mass of A	m^A	10 kg
Mass of B	m^B	20 kg
Mass of C	m^C	20 kg
Earth’s gravitational acceleration	g	$9.81 \frac{m}{s^2}$
\hat{n}_y measure of force applied to C_B	H	200 N
Angle from \hat{n}_x to \hat{a}_x with $+\hat{n}_z$ sense	q_A	Variable
Angle from \hat{n}_x to \hat{b}_x with $+\hat{n}_z$ sense	q_B	Variable
Angle from \hat{n}_x to \hat{c}_x with $+\hat{n}_z$ sense	q_C	Variable

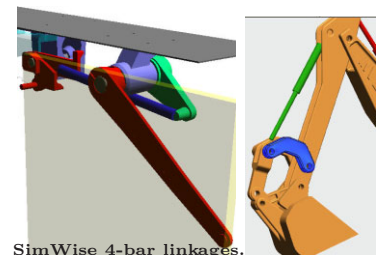
Complete the following **MG road-map** to determine this systems’s **static configuration**.

Variable	Translate/Rotate	Direction (unit vector)	System S	FBD of S	About point	MG road-map equation	Additional Unknowns
q_A	Rotate	\hat{n}_z	A, B	Draw	A_o	$\hat{a}_x \cdot \vec{M}^{S/A_o} = 0$	F_x^C, F_y^C
q_B	Rotate	\hat{n}_z	B	Draw	B_o	$\hat{a}_y \cdot \vec{M}^{B/B_o} = 0$	F_x^C, F_y^C
q_C	Rotate	\hat{n}_z	C	Draw	C_o	$\hat{a}_y \cdot \vec{M}^{C/C_o} = 0$	F_x^C, F_y^C
* Additional scalar constraint equation:				$-L_A \sin(q_A) \dot{q}_A - L_B \sin(q_B) \dot{q}_B + L_C \sin(q_C) \dot{q}_C = 0$			
* Additional scalar constraint equation:				$L_A \cos(q_A) \dot{q}_A + L_B \cos(q_B) \dot{q}_B - L_C \cos(q_C) \dot{q}_C = 0$			
q_A	Dot	$\langle N_z \rangle$	System(A, B)		A_o	MotionGenesis command ©	
q_B	Dot	$\langle N_z \rangle$	B		B_o	MotionGenesis command ©	
q_C	Dot	$\langle N_z \rangle$	C		C_o	MotionGenesis command ©	

Determine the **static equilibrium** values of q_A, q_B, q_C . Use your intuition (guess), circle the **stable** solution.

Solution 1	$q_A \approx 20.0^\circ$	$q_B \approx 71.7^\circ$	$q_C = 38.3^\circ$
Solution 2	$q_A \approx 249.3^\circ$	$q_B \approx 140.2^\circ$	$q_C = 199.1^\circ$
Solution 3	$q_A \approx 30.7^\circ$	$q_B \approx 226.1^\circ$	$q_C = 254.7^\circ$

Solution at www.MotionGenesis.com ⇒ [Get Started](#) ⇒ Four-bar linkage



SimWise 4-bar linkages. Courtesy Design Simulation Technology