Show work – except for ♣ fill-in-blanks-problems (print .pdf from www.MotionGenesis.com ⇒ Textbooks ⇒ Resources).

1.1 ♣ Solving problems – what engineers <u>do</u>.

Understanding this material results from **<u>doing</u>** problems. Many problems are guided to help you synthesize processes (imitation). You are encouraged to work by yourself or with colleagues/instructors and use the textbook's reference theory and other resources.

Confucius 500 B.C.
"I hear and I forget.
I see and I remember.
I ______ and I understand."

"By three methods we may learn wisdom:
First, by reflection, which is noblest;
Second, by imitation, which is easiest;
Third by experience, which is the bitterest."

1.2 ♣ What is a vector? (Section 2.2)

Two properties (attributes) of a vector are ______________ and ______________.

1.3 ♣ What is a zero vector? (Section 2.3)

A zero vector $\vec{0}$ has a magnitude of **0/1/2/∞**.

A zero vector $\vec{0}$ has no direction. **True/False**.

1.4 ♣ Unit vectors. (Section 2.4)

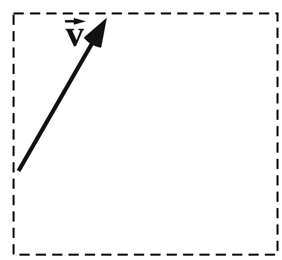
A unit vector has a magnitude of **0/1/2/∞**.

All unit vectors are equal. **True/False**.

1.5 ♣ <u>Draw</u> the following vectors: (Section 2.2)

- Long, horizontally-right vector $\vec{\mathbf{a}}$
- Short, vertically-upward vector $\vec{\mathbf{b}}$
- Outwardly-directed **<u>unit</u>** vector $\widehat{\mathbf{c}}$.

1.6 ♣ Vector magnitude and direction (orientation and sense). (Section 2.2)



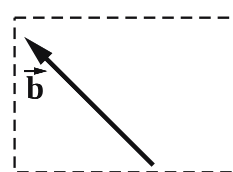
The figure to the right shows a vector $\vec{\mathbf{v}}$. **<u>Draw</u>** the following vectors.

- $\vec{\mathbf{a}}$: Same magnitude and same direction as $\vec{\mathbf{v}}$ ($\vec{\mathbf{a}} = \vec{\mathbf{v}}$).
- $\vec{\mathbf{b}}$: Same magnitude and orientation as $\vec{\mathbf{v}}$, but different sense.
- $\vec{\mathbf{c}}$: Same direction as $\vec{\mathbf{v}}$, but different magnitude.
- $\vec{\mathbf{d}}$: Same magnitude as $\vec{\mathbf{v}}$, but different direction (orientation).
- $\vec{\mathbf{e}}$: Different magnitude and different direction (orientation) as $\vec{\mathbf{v}}$.

1.7 ♣ Magnitude of a vector. (Section 2.2)

Consider a real number x and a horizontally-right pointing unit vector $\widehat{\mathbf{i}}$.
The ***magnitude*** of the vector $-x\,\widehat{\mathbf{i}}$ is (circle **<u>one</u>**): positive negative non-negative non-positive.

1.8 ♣ Negating a vector. (Section 2.8)



Complete the figure to the right by **<u>drawing</u>** the vector $-\vec{\mathbf{b}}$.

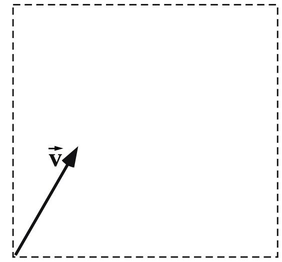
Negating the vector $\vec{\mathbf{b}}$ results in a vector with different (circle **<u>all</u>** that apply):

magnitude direction orientation sense

Historical note: Negative numbers (e.g., −3) were not widely accepted until 1800 A.D.

1.9 ♣ Multiplying a vector by a scalar. (Section 2.7)

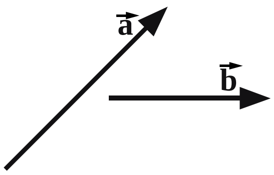
Complete the figure to the right by drawing the vectors $2\,\vec{\mathbf{v}}$ and $-2\,\vec{\mathbf{v}}$.



The following statements involve a vector $\vec{\mathbf{v}}$ and a real non-zero scalar s ($s \neq 0$).
If a statement is true, provide a numerical value for s that supports your answer

$s\,\vec{\mathbf{v}}$ can have a different ***magnitude*** than $\vec{\mathbf{v}}$.	**True/False**	$s =$
$s\,\vec{\mathbf{v}}$ can have a different ***direction*** than $\vec{\mathbf{v}}$.	**True/False**	$s =$
$s\,\vec{\mathbf{v}}$ can have a different ***sense*** than $\vec{\mathbf{v}}$.	**True/False**	$s =$
$s\,\vec{\mathbf{v}}$ can have a different ***orientation*** than $\vec{\mathbf{v}}$.	**True/False**	$s =$

1.10 ♣ Graphical vector addition/subtraction - draw. (Sections 2.6,2.8)



Draw $\vec{\mathbf{a}} + \vec{\mathbf{b}}$

Draw $\vec{\mathbf{a}} - \vec{\mathbf{b}}$

Draw $\vec{\mathbf{b}} + \vec{\mathbf{a}}$

Draw $\vec{\mathbf{b}} - \vec{\mathbf{a}}$

1.11 ♣ Visual representation of a vector dot-product. (Section 2.9)

Write the ***definition*** of the dot-product of a vector $\vec{\mathbf{a}}$ with a vector $\vec{\mathbf{b}}$.
Include a ***sketch*** with ***each symbol*** in the right-hand-side of your definition clearly labeled. The sketch should include $\vec{\mathbf{a}}$, $\vec{\mathbf{b}}$, $|\vec{\mathbf{a}}|$, $|\vec{\mathbf{b}}|$, ...

Result:

$$\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} \triangleq \underline{\qquad\qquad}$$

1.12 ♣ Visual representation of a vector cross-product. (Section 2.10)

Write the ***definition*** of the cross-product of a vector $\vec{\mathbf{a}}$ with a vector $\vec{\mathbf{b}}$.
Include a ***sketch*** with ***each symbol*** in your definition labeled and described.

Result:

$$\vec{\mathbf{a}} \times \vec{\mathbf{b}} \triangleq \underline{\qquad}\;\underline{\quad}(\theta)\;\hat{\mathbf{u}}$$

where $\hat{\mathbf{u}}$ is ______________________________

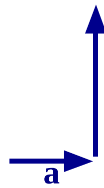
______________________ and θ is ______________________

1.13 ♣ Properties of vector dot-products and cross-products. (Sections 2.9.1 and 2.10.1)

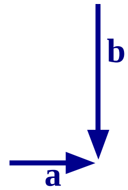
When $\vec{\mathbf{a}}$ is parallel to $\vec{\mathbf{b}}$:	$\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = 0$	**True/False**	$\vec{\mathbf{a}} \times \vec{\mathbf{b}} = \vec{\mathbf{0}}$	**True/False**
When $\vec{\mathbf{a}}$ is perpendicular to $\vec{\mathbf{b}}$:	$\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = 0$	**True/False**	$\vec{\mathbf{a}} \times \vec{\mathbf{b}} = \vec{\mathbf{0}}$	**True/False**
For arbitrary vectors $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$:	$\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = \vec{\mathbf{b}} \cdot \vec{\mathbf{a}}$	**True/False**	$\vec{\mathbf{a}} \times \vec{\mathbf{b}} = \vec{\mathbf{b}} \times \vec{\mathbf{a}}$	**True/False**

1.14 ♣ Calculating vector dot-products and cross-products via definitions. (Sections 2.9 and 2.10)

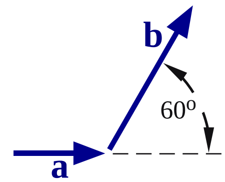
Draw a unit vector $\widehat{\mathbf{k}}$ outward-normal to the plane of the paper.
Knowing vector $\vec{\mathbf{a}}$ has magnitude 2 and vector $\vec{\mathbf{b}}$ has magnitude 4, calculate the following dot-products and cross-products via their **definitions** (2^+ significant digits).



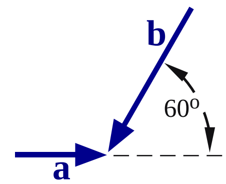
$\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} =$ ____
$\vec{\mathbf{a}} \times \vec{\mathbf{b}} =$ ____



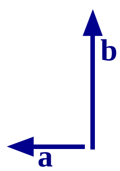
$\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} =$ ____
$\vec{\mathbf{a}} \times \vec{\mathbf{b}} =$ ____



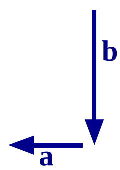
$\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} =$ ____
$\vec{\mathbf{a}} \times \vec{\mathbf{b}} =$ ____



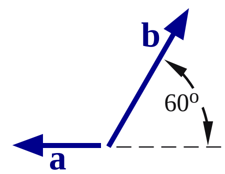
$\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} =$ ____
$\vec{\mathbf{a}} \times \vec{\mathbf{b}} =$ ____



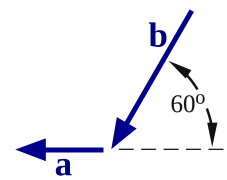
$\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} =$ ____
$\vec{\mathbf{a}} \times \vec{\mathbf{b}} =$ ____



$\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} =$ ____
$\vec{\mathbf{a}} \times \vec{\mathbf{b}} =$ ____



$\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} =$ ____
$\vec{\mathbf{a}} \times \vec{\mathbf{b}} =$ ____



$\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} =$ ____
$\vec{\mathbf{a}} \times \vec{\mathbf{b}} =$ ____

1.15 ♣ Property of scalar triple product. (Section 2.11).

For arbitrary non-zero vectors $\vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{c}}$: $\quad \vec{\mathbf{a}} \cdot (\vec{\mathbf{b}} \times \vec{\mathbf{c}}) = (\vec{\mathbf{a}} \times \vec{\mathbf{b}}) \cdot \vec{\mathbf{c}}$ **Never/Sometimes/Always**

A property of the ***scalar triple product*** is $\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} \times \vec{\mathbf{a}} = 0$. **True/False**.

1.16 Optional: Property of vector triple cross-product. (Sections 2.10.1 and 2.11)

Complete the following equation: $\vec{\mathbf{a}} \times (\vec{\mathbf{b}} \times \vec{\mathbf{c}}) = \vec{\mathbf{b}}\,(\text{\_\_\_\_}) - \vec{\mathbf{c}}\,(\text{\_\_\_\_})$

Circle true or false (show supporting work): $\vec{\mathbf{a}} \times (\vec{\mathbf{b}} \times \vec{\mathbf{c}}) = (\vec{\mathbf{a}} \times \vec{\mathbf{b}}) \times \vec{\mathbf{c}} + \vec{\mathbf{b}} \times (\vec{\mathbf{a}} \times \vec{\mathbf{c}})$ **True/False**

1.17 ♣ Optional: Proof of magnitude of vector cross product property. (Sections 2.9 and 2.10)

Letting $\widehat{\boldsymbol{\lambda}}$ be a ***unit vector*** and $\vec{\mathbf{v}}$ be ***any vector***, prove[1] $|\vec{\mathbf{v}} \times \widehat{\boldsymbol{\lambda}}|^2 = \vec{\mathbf{v}} \cdot \vec{\mathbf{v}} - (\vec{\mathbf{v}} \cdot \widehat{\boldsymbol{\lambda}})^2$.

1.18 ♣ Vector exponentiation: $\vec{\mathbf{v}}^2$ and $\vec{\mathbf{v}}^3$. Complete the 3-step proofs. (Section 2.9)

Step 1: Complete the **definition** of $\vec{\mathbf{v}}^2$ in terms of $|\vec{\mathbf{v}}|$.
Step 2: Use the **definition** of the dot-product to show how $\vec{\mathbf{v}} \cdot \vec{\mathbf{v}}$ can be expressed in terms of $|\vec{\mathbf{v}}|$.
Step 3: Combine these two definitions to provide an alternate way to calculate $\vec{\mathbf{v}}^2$ with a vector dot-product.

Result: $\vec{\mathbf{v}}^2 \triangleq |\vec{\mathbf{v}}|^{\text{\_\_}}$ $\qquad \vec{\mathbf{v}} \cdot \vec{\mathbf{v}} \underset{(2.2)}{=}$ ____ $\qquad \vec{\mathbf{v}}^2 =$ ____ $\cdot$ ____

Complete the 3-step proof that relates $\vec{\mathbf{v}}^3$ to $\vec{\mathbf{v}} \cdot \vec{\mathbf{v}}$ raised to a real number.

Result: $\vec{\mathbf{v}}^3 \triangleq |\vec{\mathbf{v}}|^{\text{\_\_}} \underset{(2.4)}{=} \left(\sqrt{\text{\_\_\_\_}}\right)^{\text{\_\_}} = (\vec{\mathbf{v}} \cdot \vec{\mathbf{v}})^{\text{\_\_}}$

[1] One way to prove this is to write $(\vec{\mathbf{v}} \times \widehat{\boldsymbol{\lambda}})^2 = (\vec{\mathbf{v}} \times \widehat{\boldsymbol{\lambda}}) \cdot (\vec{\mathbf{v}} \times \widehat{\boldsymbol{\lambda}}) \underset{(2.10)}{=} \vec{\mathbf{v}} \cdot [\widehat{\boldsymbol{\lambda}} \times (\vec{\mathbf{v}} \times \widehat{\boldsymbol{\lambda}})]$ and then use the vector triple cross-product property $\vec{\mathbf{a}} \times (\vec{\mathbf{b}} \times \vec{\mathbf{c}}) = \vec{\mathbf{b}}(\vec{\mathbf{a}} \cdot \vec{\mathbf{c}}) - \vec{\mathbf{c}}(\vec{\mathbf{a}} \cdot \vec{\mathbf{b}})$ from Section 2.10. Alternately, it is helpful to write $\vec{\mathbf{v}} = v_\perp \widehat{\boldsymbol{\lambda}}_\perp + v_{||} \widehat{\boldsymbol{\lambda}}$ where $v_\perp \widehat{\boldsymbol{\lambda}}_\perp$ is the component of $\vec{\mathbf{v}}$ that is perpendicular to $\widehat{\boldsymbol{\lambda}}$ and $v_{||} \widehat{\boldsymbol{\lambda}}$ is the component of $\vec{\mathbf{v}}$ that is parallel to $\widehat{\boldsymbol{\lambda}}$.

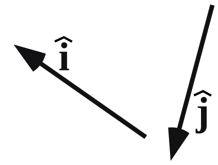
1.19 ♣ $|c\,\hat{\mathbf{a}}_x|$ **Calculate vector magnitude with dot products.** (Section 2.9 and Hw 1.18)

Show how the vector dot-product can be used to show that the magnitude of the vector $c\,\hat{\mathbf{a}}_x$ (c is a positive or **negative** number and $\hat{\mathbf{a}}_x$ is a unit vector) can be written solely in terms of c (without $\hat{\mathbf{a}}_x$).

Result: $|c\,\hat{\mathbf{a}}_x| = \sqrt{\square \cdot \square} = \sqrt{c^2 * \square \cdot \square} = \sqrt{\square} = \text{abs}(c)$

1.20 † **Magnitude of the vector $\vec{\mathbf{v}}$.** Show work. (Section 2.9)

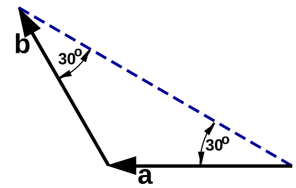
Knowing the angle between a unit vector $\hat{\mathbf{i}}$ and unit vector $\hat{\mathbf{j}}$ is 110°, calculate a numerical value for the magnitude of $\vec{\mathbf{v}} = 3\,\hat{\mathbf{i}} + 4\,\hat{\mathbf{j}}$.



Result: $|\vec{\mathbf{v}}| \approx \boxed{4.098}$ Note: The answer is *not* 5.

1.21 ♣ **Angle between vectors.** (Section 2.9)

Referring to the figure to the right, find the numerical value for the angle between vector $\vec{\mathbf{a}}$ and vector $\vec{\mathbf{b}}$.



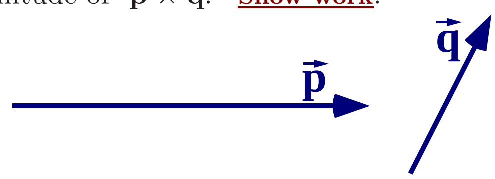
$\angle(\vec{\mathbf{a}}, \vec{\mathbf{b}}) = \square°$

1.22 **Visual estimation of vector dot/cross-products.** Show work. (Sections 2.9 and 2.10)

Estimate (e.g., using your pinky) the magnitude of the vector $\vec{\mathbf{p}}$ shown below. Note: 1 inch $\triangleq$ 2.54 cm.
Estimate the angle between $\vec{\mathbf{p}}$ and $\vec{\mathbf{q}}$, $\vec{\mathbf{p}} \cdot \vec{\mathbf{q}}$, and the magnitude of $\vec{\mathbf{p}} \times \vec{\mathbf{q}}$. Show work.

Result: (Provide numerical results with 1 or more significant digits).

$\lvert\vec{\mathbf{p}}\rvert \approx \square$ cm	$\angle(\vec{\mathbf{p}}, \vec{\mathbf{q}}) \approx \square°$
$\vec{\mathbf{p}} \cdot \vec{\mathbf{q}} \approx \square\ \text{cm}^2$	$\lvert\vec{\mathbf{p}} \times \vec{\mathbf{q}}\rvert \approx \square\ \text{cm}^2$



1.23 ♣ **Form the *unit* vector $\hat{\mathbf{u}}$ having the same direction as $c\,\hat{\mathbf{a}}_x$.** (Section 2.4)

Result: $\hat{\mathbf{u}} = \square\ \hat{\mathbf{a}}_x$ Note: $\hat{\mathbf{a}}_x$ is a unit vector and c is a non-zero real number, e.g., 3 or −3

1.24 ♣ **Coefficient of $\hat{\mathbf{u}}$ in cross products – definitions and trig functions.** (Section 2.10)

The ***cross product*** of vectors $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ can be written in terms of a real scalar s as $\vec{\mathbf{a}} \times \vec{\mathbf{b}} = s\,\hat{\mathbf{u}}$ where $\hat{\mathbf{u}}$ is a unit vector perpendicular to both $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ in a direction defined by the **right-hand rule**. The coefficient s of the unit vector $\hat{\mathbf{u}}$ is inherently non-negative. **True/False**.

1.25 ♣ **Orthogonal vectors: Insights via drawing.** (Section 2.10)

Consider three unit vectors $\hat{\mathbf{a}}$, $\hat{\mathbf{b}}$, and $\hat{\mathbf{c}}$.
Vector $\hat{\mathbf{a}}$ is perpendicular to vector $\hat{\mathbf{b}}$.
Vector $\hat{\mathbf{b}}$ is perpendicular to vector $\hat{\mathbf{c}}$.
Vector $\hat{\mathbf{a}}$ is not parallel to vector $\hat{\mathbf{c}}$.

In **all** cases, $\hat{\mathbf{a}}$ is perpendicular to $\hat{\mathbf{c}}$. **True/False**.
Explain your answer by drawing $\hat{\mathbf{a}}$, $\hat{\mathbf{b}}$, $\hat{\mathbf{c}}$ and relevant angles.

1.26 **Calculating distance between a point and a line via cross-products.** (Section 2.10.2)

Draw a horizontally-right unit vector $\hat{\mathbf{a}}_x$ and vertically-upward unit vector $\hat{\mathbf{a}}_y$.
Draw a point Q whose position vector from a point P is $\vec{\mathbf{r}}^{\,Q/P} = 5\,\hat{\mathbf{a}}_x$.
Draw a line L that passes through point P and is parallel to $\hat{\mathbf{u}} = \frac{3}{5}\,\hat{\mathbf{a}}_x + \frac{4}{5}\,\hat{\mathbf{a}}_y$.
Calculate the ***distance*** d between Q and L using both formulas in equation (2.9).

Result: $\text{d} \underset{(2.9)}{=} \square = \boxed{4}$ $\text{d} \underset{(2.9)}{=} \square = \boxed{4}$

1.27 ♣ Vector operations and units. **(Chapter 2)**

Circle the vector operations below (scalar multiplication, addition, dot-product, etc.) that are **defined** for a position vector $\vec{\mathbf{a}}$ (with **units** of m) and a velocity vector $\vec{\mathbf{b}}$ (with **units** of $\frac{\text{m}}{\text{s}}$).

$$-\,\vec{\mathbf{a}} \qquad 5\,\vec{\mathbf{a}} \qquad \vec{\mathbf{a}}\,/\,5 \qquad \vec{\mathbf{a}} + \vec{\mathbf{b}} \qquad \vec{\mathbf{a}} \cdot \vec{\mathbf{b}} \qquad \vec{\mathbf{a}} \times \vec{\mathbf{b}}$$

1.28 ♣ "Popular" vector operations. **(Chapter 2)**

For each vector operation, provide its name and determine whether it produces a scalar or vector.

Name	Symbol	Example	Operation produces
Addition	$+$	$\vec{\mathbf{a}} + \vec{\mathbf{b}}$	**Scalar/Vector**
___	$-$	$-\vec{\mathbf{b}}$	**Scalar/Vector**
___	$*$	$3 * \vec{\mathbf{b}}$	**Scalar/Vector**
___	$\cdot$	$\vec{\mathbf{a}} \cdot \vec{\mathbf{b}}$	**Scalar/Vector**
___	$\times$	$\vec{\mathbf{a}} \times \vec{\mathbf{b}}$	**Scalar/Vector**

1.29 ♣ Using vector identities to simplify expressions **(refer to Homework 1.13).**

One reason to treat vectors as ***basis-independent*** quantities is to simplify vector expressions ***without*** resolving the vectors into orthogonal "$\vec{\mathbf{x}}, \vec{\mathbf{y}}, \vec{\mathbf{z}}$" or "$\vec{\mathbf{i}}, \vec{\mathbf{j}}, \vec{\mathbf{k}}$" components. Simplify the following vector expressions using various properties of dot-products and cross-products.

Express your results in terms of dot-products $\cdot$ and cross-products $\times$ of the arbitrary vectors $\vec{\mathbf{u}}$, $\vec{\mathbf{v}}$, $\vec{\mathbf{w}}$ (i.e., $\vec{\mathbf{u}}$, $\vec{\mathbf{v}}$, $\vec{\mathbf{w}}$ are not orthogonal).

Vector expression	Simplified vector expression
$(3\,\vec{\mathbf{u}} - 2\,\vec{\mathbf{v}}) \times (\vec{\mathbf{u}} + \vec{\mathbf{v}})$	___ $\vec{\mathbf{u}} \times \vec{\mathbf{v}}$
$(3\,\vec{\mathbf{u}} - 2\,\vec{\mathbf{v}}) \cdot (\vec{\mathbf{u}} + \vec{\mathbf{v}})$	___ $\vec{\mathbf{u}}^2$ $-$ ___ $\vec{\mathbf{v}}^2$ $+$ ___ $\vec{\mathbf{u}} \cdot \vec{\mathbf{v}}$
$(\vec{\mathbf{u}} - \vec{\mathbf{v}}) \cdot (\vec{\mathbf{u}} + \vec{\mathbf{v}})$	___ $-$ ___
$(3\,\vec{\mathbf{u}} - 2\,\vec{\mathbf{v}}) \times (\vec{\mathbf{u}} + \vec{\mathbf{v}}) \cdot (2\,\vec{\mathbf{u}} - 7\,\vec{\mathbf{v}})$	___
$(\vec{\mathbf{u}} + \vec{\mathbf{v}}) \times (\vec{\mathbf{v}} + 2\,\vec{\mathbf{w}}) \cdot (\vec{\mathbf{w}} + 2\,\vec{\mathbf{u}})$	___ $\vec{\mathbf{u}} \times \vec{\mathbf{v}} \cdot \vec{\mathbf{w}}$

1.30 Changing a vector equation to scalar equations. **Show work.** **(Section 2.9.5)**

Draw three mutually orthogonal unit vectors $\widehat{\mathbf{p}}$, $\widehat{\mathbf{q}}$, $\widehat{\mathbf{r}}$.

(a) Use a vector operation (e.g., $+$, $-$, $*$, $\cdot$, $\times$) to transform the following **vector** equation into **one scalar** equation and subsequently solve the scalar equation.

$$(2x - 4)\,\widehat{\mathbf{p}} = \vec{\mathbf{0}} \quad \overset{??}{\Rightarrow} \quad x = 2$$

(b) Show ***every*** vector operation (e.g., $+$, $-$, $*$, $\cdot$, or $\times$) that transforms the following **vector** equation into **three scalar** equations and subsequently solve the scalar equations for x, y, z.

$$(2\,x - 4)\,\widehat{\mathbf{p}} + (3\,y - 9)\,\widehat{\mathbf{q}} + (4\,z - 16)\,\widehat{\mathbf{r}} = \vec{\mathbf{0}}$$

Result: $x = 2 \qquad y = 3 \qquad z =$ ___

1.31 ♣ Number of independent scalar equations from one vector equation. (Section 2.9.5)

Consider the **vector** equation shown to the right that can be useful for static analyses of any system S.

$$\vec{\mathbf{F}}^{S} = \vec{\mathbf{0}}$$

Complete the blanks in the table to the right with **all** integers that could be equal to the number of ***independent scalar*** equations produced by the previous vector equation for any system S.

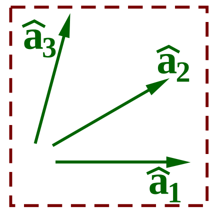
Hint: See Homework 1.30 for ideas.

System type	Integer(s)
1D (line)	0, ____
2D (planar)	0, ____
3D (spatial)	0, ____

Note: Regard 1D/linear as meaning $\vec{\mathbf{F}}^{S}$ can be expressed in terms of a single unit vector $\hat{\mathbf{i}}$ whereas 2D/planar means $\vec{\mathbf{F}}^{S}$ can be expressed in terms of two non-parallel unit vectors $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$, and 3D/spatial means $\vec{\mathbf{F}}^{S}$ can be expressed in terms of three non-coplanar unit vectors $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, $\hat{\mathbf{k}}$.

1.32 ♣ Vector concepts: Solving a vector equation (just circle true or false and fill-in the blank).

Consider the following vector equation written in terms of the scalars x, y, z and three unique non-orthogonal ***coplanar*** unit vectors $\hat{\mathbf{a}}_1$, $\hat{\mathbf{a}}_2$, $\hat{\mathbf{a}}_3$.



$$(2\,x - 4)\;\hat{\mathbf{a}}_1 \;+\; (3\,y - 9)\;\hat{\mathbf{a}}_2 \;+\; (4\,z - 16)\;\hat{\mathbf{a}}_3 \;=\; \vec{\mathbf{0}}$$

The **unique** solution to this vector equation is $x = 2,\; y = 3,\; z = 4$. **True/False**.

Explain: $\hat{\mathbf{a}}_2$ can be expressed in terms of $\hat{\mathbf{a}}_1$ and $\hat{\mathbf{a}}_3$ (i.e., $\hat{\mathbf{a}}_2$ is a linear combination of $\hat{\mathbf{a}}_1$ and $\hat{\mathbf{a}}_3$). Hence the vector equation produces ____ linearly independent scalar equations.

1.33 ♣ A vector revolution in geometry. (Chapter 2)

The relatively new invention of vectors (Gibbs ≈ 1900 AD) has revolutionized Euclidean geometry (Euclid ≈ 300 BC). For each geometrical quantity below, circle the vector operation(s) (either the dot-product, cross-product, or both) that is ***most*** useful for their calculation.

Length: $\cdot$ $\times$	Angle: $\cdot$ $\times$
Area: $\cdot$ $\times$	Volume: $\boxed{\cdot}$ $\boxed{\times}$

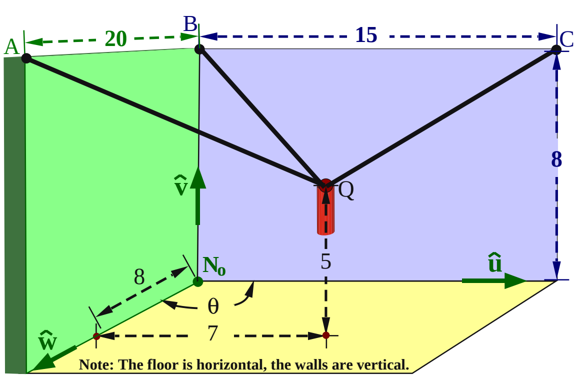
1.34 † Microphone cable lengths (non-orthogonal walls) **"It's just geometry".** Show work.

A microphone Q is attached to three pegs A, B, C by three cables. Knowing the peg locations, microphone location, and the angle θ between the vertical walls, express L_A, L_B, L_C solely in terms of numbers and θ. Next, complete the table by calculating L_B when $\theta = 120°$.

Hint: To do this **efficiently**, use only unit vectors $\hat{\mathbf{u}}$, $\hat{\mathbf{v}}$, $\hat{\mathbf{w}}$, and do **not** introduce an **orthogonal** set of unit vectors.

Hint: Use the distributive property of the vector dot-product as shown in Section 2.9.1 and Homework 2.4.

Note: Synthesis problems are difficult. Engineers solve problems. Think, talk, draw, sleep, walk, get help, ...



Distance between A and B	20 m
Distance between B and C	15 m
Distance between N_o and B	8 m
Distance along back wall (see picture)	7 m
Q's height above N_o	5 m
Distance along side wall (see picture)	8 m
L_A: Length of cable joining A and Q	16.9 m
L_B: Length of cable joining B and Q	8.1 m
L_C: Length of cable joining C and Q	14.2 m

$$\vec{\mathbf{r}}^{\,Q/N_o} = 7\,\hat{\mathbf{u}} + 5\,\hat{\mathbf{v}} + 8\,\hat{\mathbf{w}}$$

Result:

$L_A = \sqrt{202 - 168\cos(\theta)}$ $\qquad$ $L_B =$ ____ $\qquad$ $L_C = \sqrt{137 - \_\_\_\_ \cos(\theta)}$